Twistors and the Superstring

Nathan Berkovits

ICTP South American Institute for Fundamental Research

IFT-UNESP, São Paulo

arXiv:1604.04617

"Untwisting the Pure Spinor Formalism to the RNS and Twistor String in a Flat and $AdS_5 \times S^5$ Background"

N=4 d=4 twistor superstring

$$S = \int d^2z[Y_A(\bar{\nabla} + \mathcal{A})Z^A] + S_{current}$$

$$+right - movers$$

$$= \int d^2z[Y_A\bar{\partial}Z^A + b\bar{\partial}c + \tilde{b}\bar{\partial}\tilde{c}] + S_{current}$$

$$+right - movers$$

$$Q = \int dz[cT + \tilde{c}(Y_A Z^A) + bc\partial c]$$

$$V = c\Phi^I(Z)J^I_{current}$$

$$= c[\phi_+(\lambda,\mu) + \eta^j\psi_j(\lambda,\mu) + \eta^j\eta^k\phi_{jk}(\lambda,\mu)$$

$$+(\eta)^3_j\psi^j(\lambda,\mu) + (\eta)^4\phi_-(\lambda,\mu)]^I J^I_{current}$$

D=10 pure spinor superstring

$$S = \int d^2z (\frac{1}{2}\partial x^m \bar{\partial} x_m + p_\alpha \bar{\partial}\theta^\alpha + w_\alpha \bar{\partial}\lambda^\alpha + \widehat{p}_\alpha \partial\widehat{\theta}^\alpha + \widehat{w}_\alpha \partial\widehat{\lambda}^\alpha)$$

$$\pi^m = \partial x^m - \frac{1}{2}\partial\theta\gamma^m\theta, \quad d_\alpha = p_\alpha - \frac{1}{2}\partial x_m(\gamma^m\theta)_\alpha - \frac{1}{8}(\theta\gamma^m\partial\theta)(\gamma_m\theta)_\alpha$$

$$Q = \int dz G^+ = \int dz \lambda^\alpha d_\alpha, \qquad V = \lambda^\alpha A_\alpha(x,\theta)$$

$$b = G^- = -w_\alpha\partial\theta^\alpha + \frac{1}{2(\lambda\bar{\lambda})}[(\pi^m(\bar{\lambda}\gamma_m d) + (w\gamma_m\bar{\lambda})(\lambda\gamma^m\partial\theta)]$$

D=10 twistor string

$$S = \int d^2z d^2\kappa [-\Phi_\alpha \bar{D}\Theta^\alpha + \hat{\Phi}_\alpha D\hat{\Theta}^\alpha - \frac{1}{8}(\Theta\gamma^m D\Theta)(\hat{\Theta}\gamma_m \bar{D}\hat{\Theta})]$$

$$= \int d^2z [d_\alpha \bar{\partial}\theta^\alpha + \hat{d}_\alpha \partial\hat{\theta}^\alpha + w_\alpha \bar{\partial}\lambda^\alpha + \hat{w}_\alpha \partial\hat{\lambda}^\alpha + \mu_\alpha \bar{\partial}\nu^\alpha + \hat{\mu}_\alpha \partial\hat{\nu}^\alpha$$

$$-\frac{1}{2}(\nu\gamma^m\lambda - \frac{1}{2}\theta\gamma^m\partial\theta)(\hat{\nu}\gamma_m\hat{\lambda} - \frac{1}{2}\hat{\theta}\gamma_m\bar{\partial}\hat{\theta})]$$

$AdS_5 \times S^5$ twistor string

$$S = r^2 \int d^2z d\kappa d\bar{\kappa} (G^{-1} D G)^J_R (G^{-1} \bar{D} G)^R_J$$

$$= r^2 \int d^2z d\kappa d\bar{\kappa} (D\Theta^J_R \bar{D}\Theta^R_J + D\Theta^J_R \Theta^R_K \bar{D}\Theta^K_S \Theta^S_J)$$

$$= r^2 \int d^2z [(g^{-1}\partial g)^J_R (g^{-1}\bar{\partial} g)^R_J$$

$$+\Lambda^J_R (\bar{\nabla}\Lambda)^R_J + \widehat{\Lambda}^J_R (\nabla\widehat{\Lambda})^R_J + \Lambda^J_R \Lambda^R_K \widehat{\Lambda}^K_S \widehat{\Lambda}^S_J - \Lambda^R_J \Lambda^J_S \widehat{\Lambda}^S_K \widehat{\Lambda}^K_R]$$

$$= r^2 \int d^2z [(g^{-1}\partial g)^J_R (g^{-1}\bar{\partial} g)^R_J$$

$$+Y_A (\bar{\nabla} Z)^A + \widehat{Y}_A (\nabla \widehat{Z})^A + R_{MNPQ} (Y\gamma^{MN} Z)(\widehat{Y}\gamma^{PQ}\widehat{Z})]$$