Quantum Gravity in superspace

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The Wheeler-deWitt equation

In canonical GR, the Hamiltonian is made out of local constraints.

$$\mathcal{H} = \int_{M^3} d^3x \left(N(x)H(x) + N^i(x)H_i(x) \right)$$

$H_i(x)$ encodes spatial diffeomorphisms.

$H(x)$ is identified with "refoliations." It gives rise to:

$$\hat{H}\Psi = \left(G_{ijkl} \frac{\delta^2}{\delta g_{ij} \delta g_{kl}} + \sqrt{g} \left(-R(g) + 2\Lambda \right) \right) \Psi[g_{ij}] = 0$$

Many technical problems. But also conceptual: e.g.: observables should commute with Hamiltonian.

$\underbrace{\text{Only histories have meaning}}_{GR}$ vs $\underbrace{\text{measurements are instantaneous}}_{QM}$

Summary of non-perturbative issues I deem important.

Don't mix well:

- Quantum mechanical notions
  1. Instantaneous collapse of the wavefunction.
  2. Non-gauge evolution.
  3. Quantum superposition principle.
- General relativistic notions
  1. Space-time covariance.
  2. A Hamiltonian which mingles local gauge symmetries and evolution.
  3. A fixed causal structure.

Still leave out quantum cosmology and measurement problem.

Simpler symmetries

Configuration space of ADM is $\mathcal{Q}$ =Riem(M). But refoliations don't act pointwise in Riem (don't form group orbits).

- Demand symmetries $\mathcal{G}$ act locally in M and pointwise in $\mathcal{Q}$.

$\Rightarrow$ Generators linear in momenta, with well-defined $\mathcal{Q}/\mathcal{G}$.

  - Spatial diffeomorphisms ($\nabla^a \pi^b_a = 0$)
  - Spatial Weyl ($\pi_{ab} g^{ab} = 0$)

Theory should be at most invariant wrt these transfs.

Shape dynamics[HG, Gryb, Koslowski 11; HG, Koslowski 12; Mercati 15] is such a theory:

- Disentangles dynamics from symmetries.
- Matches GR in a broad set of circumstances. But not always.
- Has a non-local global Hamiltonian.

The static wavefunction

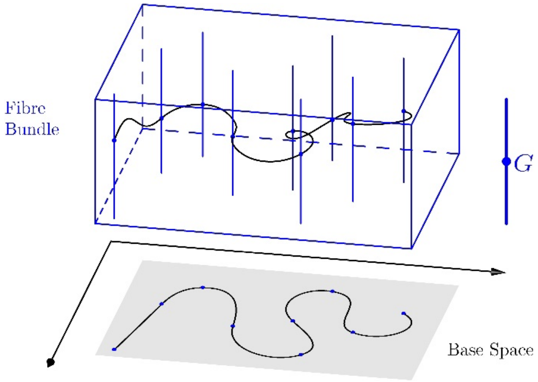
Given an action functional on curves on $\mathcal{Q}$, $S(\gamma)$, for $\gamma : I \to \mathcal{Q}$, respecting the given gauge symmetry group $\mathcal{G}$. Defines $\tilde{S}([\gamma])$, for $[\gamma] : I \to \mathcal{Q}/\mathcal{G}$. Can define a propagator:

$$W(g_1, [g_2]) := A \int_{g_1}^{[g_2]} \mathcal{D}[\gamma] \exp\left[i\tilde{S}[\gamma(\lambda)]/\hbar\right]$$

Measure is projected Liouville.[Barvinsky '91]

The static wavefunction

$\tilde{S}([\gamma]) := S(\gamma_H)$ (horizontal lift wrt functional connection-form)



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As in Hartle-Hawking,[Hartle, Hawking '83], Define static probability density:

$$P(g) = |W(g^*, g)|^2 = |\Psi(g)|^2$$

but now g^* is "most homogeneous configuration" [HG '16]

$$g^* = \text{Arg}(\text{Sup}_{g\in\mathcal{Q}} \dim(\text{Iso}_g))$$

Is unique for $M = S^3$ and $\mathcal{G} = \text{Diff} \ltimes \mathcal{C} \Rightarrow g^* = d\Omega_o^3$. [Fischer, Moncrief '97]

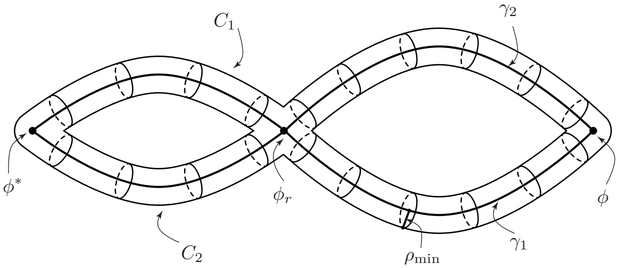
Records and conservation of probability [HG '16]

g has a semi-classical record of g_r iff

$\forall \gamma^\alpha : I \to \mathcal{Q} \mid \{S'_{|\gamma^\alpha} = 0, S(\gamma^\alpha) >> \hbar, \ \gamma^\alpha(0) = g^*, \gamma(1) = g\}$

then $\exists! t^\alpha \in I \mid \gamma^\alpha(t_\alpha) = g_r$.

Records and conservation of probability [HG '16]



(wouldn't work with refoliation invariance)

Records and conservation of probability [HG '16]

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then $\exists! t^\alpha \in I \mid \gamma^\alpha(t_\alpha) = g_r$.

$$W_{\text{cl}}(g^*, g) = W_{\text{cl}}(g^*, g_r)W_{\text{cl}}(g_r, g) + \mathcal{O}(\hbar^2)$$

Can show that, for $\mathcal{Q}_{(r)} := \{g \,|\, P(g) = P(g_r)P(g_r, g)\}$
a choice of subset for which there is no redundancy of records:

$$\mathcal{S}_{(r)} := \{g_i \in \mathcal{Q}_r,\ i \in \mathcal{I} |\, P(g_i) \neq P(g_j)P(g_j, g_i)\ \forall i, j \in \mathcal{I}\}$$

then $P(\mathcal{S}_{(r)}) = \sum_{g\in\mathcal{S}_{(r)}} |\Psi(g)|^2 \leq |\Psi(g_r)|^2 = P(g_r)$

In this sense there is conservation of probabilities.

Geometrical toy model (‘free-particle in Riem’)

For now only with $\mathcal{G}$ =Diff(M), but still $g^* = d\Omega^3$.

$$S[\gamma] = \int dt \langle \dot{\gamma}, \dot{\gamma} \rangle^{1/2}_{\gamma(t)} = \int dt \left(\int d^3x \, \dot{\gamma}_{ab} \, \gamma^{ac} \, \gamma^{bd} \, \dot{\gamma}_{cd} \sqrt{g} \right)^{1/2}$$

Horizontal lift given by orthogonality wrt fibers (not a gauge-section).[Vilkowisky ‘77, DeWitt ‘94, HG ‘10]

Geodesics explicit, in closed form.[Freed, Groisser ‘89; Michor, Medrano ‘91]

Super-Riemman curvature:

$$R(\mathbf{h}_1, \mathbf{h}_2)\mathbf{h}_3 = -\frac{1}{4}[[\mathbf{h}_1, \mathbf{h}_2], \mathbf{h}_3] + \frac{3}{16}\left(\mathrm{tr}_g(\mathbf{h}_1\mathbf{h}_3)\mathbf{h}_2 - \mathrm{tr}_g(\mathbf{h}_2\mathbf{h}_3)\mathbf{h}_1\right)$$

For $h_{ab}g^{ab} = 0$ (otherwise zero).

Geodesics don’t reconverge $\Rightarrow$ no interference terms in semi-classical approximation.

Semi-classical transition amplitudes

$$|W_{\rm cl}(g_i,g_f)|^2 \propto \sum_{\alpha\in I}\Delta_{\gamma^\alpha_{\rm cl}} + \underbrace{2\sum_{\alpha\neq\alpha'}|\Delta_{\gamma^\alpha_{\rm cl}}\Delta_{\gamma^{\alpha'}_{\rm cl}}|^{1/2}\cos\left(\frac{S_{\gamma^\alpha_{\rm cl}}-S_{\gamma^{\alpha'}_{\rm cl}}}{\hbar}\right)}_{\text{interference}}$$

where $\Delta^\alpha_{\gamma_{\rm cl}}(g_i,g_f) := \det\left(-\frac{\delta^2 S(\gamma^\alpha_{\rm cl})}{\delta g_i\delta g_f}\right)$ (measures spread of trajectories around γ.)

For finite-dimensions and geodesic action:[Visser '93]

$$\Delta^{\text{small d}}_\gamma(x,y) = 1+\frac{1}{6}(R_{ab}\dot\gamma^a\dot\gamma^b S[\gamma]^2+\mathcal{O}(S[\gamma])^3)$$

Long story about how to translate this to infinite-d.[Michor et al '01, HG '16]

Relative probabilities

For two different ranges of initial transverse traceless initial directions, H_1, H_2 at $d\Omega^3$

$$\frac{\int_{H_1} \mathcal{D}h^1 J_{TT}(g^0) P(g(\epsilon))}{\int_{H_2} \mathcal{D}h^2 J_{TT}(g^0) P(g(\epsilon))} = 1 +$$

$$\frac{1}{4} \frac{\int_{H_2} \mathcal{D}h^2 \left(\int d^3x \sqrt{g^*}\, h_{ab} g_*^{ac} g_*^{bd} h_{cd} \right) - \int_{H_1} \mathcal{D}h^1 \left(\int d^3x \sqrt{g^*}\, h_{ab} g_*^{ac} g_*^{bd} h_{cd} \right)}{V_H} \epsilon^2 + \ldots$$

This is the relative probability for two screens defined by the same arc-length distance, in an expansion for small arc-length distance ϵ from g^0.

Can be calculated explicitly for given choices of ranges H_1 and H_2 in terms of eigenvalues for a basis of TT-tensors on $d\Omega^3$. [Gerlach '78]

Summary of non-perturbative issues I deem important.

Mix better:

- Quantum mechanical notions
  1. No collapse of the wavefunction (static prob density).
  2. Non-gauge evolution (semi-classical approx).
  3. Quantum superposition principle.
- Geometry in Riem
  1. Refoliation not fundamental (recovered relationally).
  2. A Hamiltonian which separates local gauge symmetries and global evolution.
  3. Causal structures corresponding to different extremal curves (can interfere).

Challenge: recover standard GR.