Symmetries in Quantum Field Theory and Gravity

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This is ok as far as it goes, but what about discrete symmetries? This argument seems to put a bound on the order of a discrete global symmetry, but how about Z_2?

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It would be nice to have arguments that avoid these caveats, and anyways it is good to understand these issues better!

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(3) For any $\mathcal{O}(x) \in \mathcal{L}_x$, the set of local operators at $x \in \Sigma \times \mathbb{R}$, we have

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(4) For any open subregion $R \subseteq \Sigma$ and any $g \in G$, there exists a unitary operator $U(g, R)$ such that

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More generally, we will say that a symmetry which in addition to (1) – (3) also obeys (4) – (5) is *splittable*.

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Let U be a unitary operator on a Hilbert space $\mathcal{H} = \bigotimes_i \mathcal{H}_i$, with the property that it sends any operator which acts nontrivially only on one $\mathcal{H}_i$ to another such operator on the same $\mathcal{H}_i$. Then $U = \bigotimes_i U_i$.

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In fact the existence of $U(g, R)$ can be proven formally within algebraic quantum field theory, provided one assumes the theory possesses a "splitting property" that is essentially a continuum version of the factorizability of the Hilbert space. Buchholz/Doplicher/Longo

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- For continuous gauge groups this requires massless gauge bosons, while for discrete gauge groups it implies the existence of a non-trivial topological field theory at sufficiently low energies. In either case finite-energy charged excitations are allowed in infinite volume.
- If we have a gauge theory in the bulk which is not in its Coulomb phase, for example in a confining or Higgs phase, then we do not expect it to imply a global symmetry in the dual CFT.

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Gauge transformations act on these as

$$\hat{L}'_g = \hat{L}_{h_+^{-1} g h_+}$$
$$\hat{R}'_g = \hat{R}_{h_-^{-1} g h_-}$$
$$\hat{U}^{\alpha\prime}_{ij} = \sum_{kl} D^{\alpha}_{ik}(h_+)\hat{U}^{\alpha}_{kl} D^{\alpha}_{lj}\left(h_-^{-1}\right).$$

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When we have charged matter fields, there are also gauge-invariant operators obtained by stretching Wilson lines between them, for example

$$\phi^{\dagger}(x) P e^{\int_x^y A} \phi(y).$$

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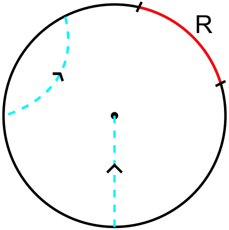

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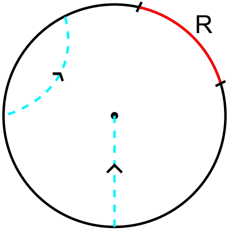

Note in particular the localized boundary symmetry operator

$$U(g, R) \equiv \prod_{l \in R} L_g(l).$$

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Claim: these are inconsistent.

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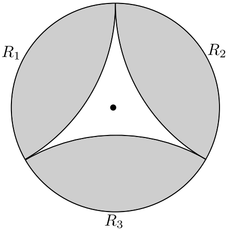

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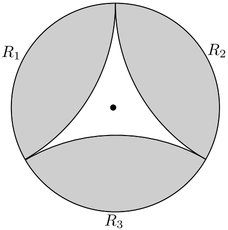

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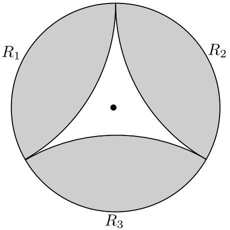

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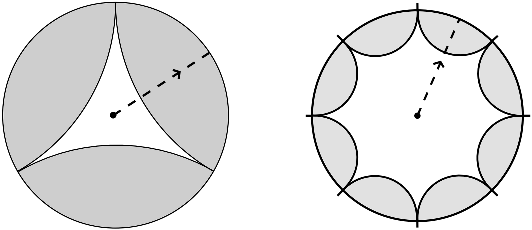
Since each $U(g, R_i)$ is localized in the boundary, it can only affect the bulk within the "entanglement wedge" of R_i. Since our charged operator is not in the entanglement wedge for any R_i, it must commute with all the $U(g, R_i)$ (and also U_{edge}). But then it must also commute with $U(g, \Sigma)$, which contradicts the assumption that the object is charged!

This contradiction is easily avoided if we instead consider a *gauge* symmetry in the bulk: any charged operator then requires a Wilson line attaching it to the boundary, and this can be detected by the $U(g, R_i)$ for whichever R_i the Wilson line ends in.



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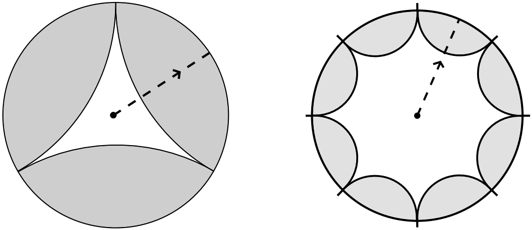

Note that even if the object has finite size, by increasing the number of regions we can pull the entanglement wedges back to the boundary, so indeed this dressing really needs to make it all the way to infinity.

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- The operators that create charged objects become charged local operators on the boundary.
- The algebra of these two is controlled by the algebra of the Wilson line and L_g.
- This establishes that these $U(g, R)$ obey (1-5), up to showing that the charged operators transform in a faithful representation of the bulk gauge group (I'll do this in a moment).

Conversely, if we assume that there is a global symmetry in the boundary theory then the $U(g, R)$'s give boundary conditions for a bulk gauge field, whose bulk equation of motion can then be solved (assuming a local semiclassical description with some effective action) to reconstruct the full set of surface operators in the bulk.

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- If G is continuous then we can use $U(g,R)$ to extract the Noether current J_μ, which is a rescaling of $F_{\mu r}$ at the boundary.
- The boundary conditions are less clear for general discrete groups, but for $\mathbb{Z}_p$ we can use the Banks-Seiberg Lagrangian

$$\mathcal{L} = \frac{ip}{2\pi}\int B_{d-1}\wedge dA_1$$

to observe that

$$U(\theta,R) = e^{i\theta\int_R B},$$

which gives the boundary conditions for B (A is its canonical conjugate).

States

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Theorem

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There is an elegant proof of this based on characters:Levy consider the representation

$$\rho_n \equiv (1 \oplus \rho \oplus \rho^* \oplus \rho \otimes \rho^*)^n,$$

which has character

$$\chi_n = |1 + \chi_\rho|^{2n}.$$

We can compute the number of times an irreducible representation appears within ρ_n via

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attaining the maximum only when $g=e$ since ρ is faithful. But then we have

$$\lim_{n\to\infty}\frac{\int_G dg\chi_\alpha(g)|1+\chi_\rho(g)|^{2n}}{\int_G dg|1+\chi_\rho(g)|^{2n}} = d_\alpha,$$

so at some sufficiently large n we must have

$$\int_G dg\chi_\alpha(g)|1+\chi_\rho(g)|^{2n} \neq 0.$$

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Ruling this out also will complete our above argument that a gauge theory in the bulk with gauge group G implies a global symmetry G in the boundary theory.

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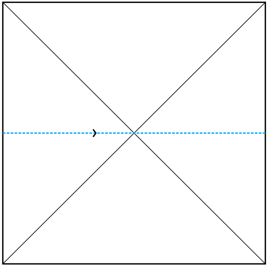
Consider the two-sided AdS-Schwarzschild geometry:



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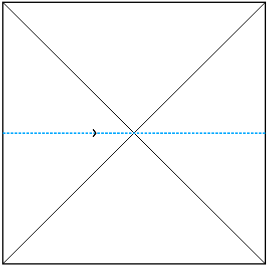

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For any g there is some irreducible representation α for which $D_\alpha(g)$ is nontrivial, so we see that $U_R(g,\Sigma)$ must act nontrivially for all g, and thus act faithfully on the single-CFT Hilbert space.

Our second argument is less general, it works only when G is connected, but it provides a useful alternative perspective.

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- This is related to the set of charged operators which exist in the CFT, since their correlation functions must be single-valued around any Dirac strings in the background gauge field.
- Since these background gauge fields are boundary conditions for the bulk gauge field, this relates the topology of the bulk gauge group to the set of charged local operators in just the right way to ensure that the bulk gauge group is represented faithfully on the CFT Hilbert space.

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Instead, I will give a rather general condition on conformal field theories which necessarily implies that any symmetry group will be compact. This will make no reference to holography.

I claim that any "reasonable" CFT should have an operator algebra which is finitely generated in the following sense:

- There exist a finite set of primary operators $\mathbb{O}_i$ such that any other primary operator eventually appears in their iterated OPEs.

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Moreover I will restrict to theories for which the thermal partition function on $\mathbb{S}^{d-1} \times \mathbb{S}^1$ is finite, which implies that the Hilbert space on $\mathbb{S}^{d-1}$ is a direct sum over finite-dimensional unitary and irreducible representations of the symmetry group G.

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Finite generation of the operator algebra then implies that there will some finite set of these representations which is faithful and generates all the others; I'll call this representation $\rho : G \mapsto U(N)$.

Note that there ARE noncompact groups with finite-dimensional faithful unitary representations, for example $\mathbb{R}$ has the representation $(e^{ix}, e^{i\sqrt{2}x})$. So we need to work harder to exclude them.

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- In fact so will the correlation functions of the rest of the operators, since they are generated by products of these ones. So we can extend the representation $U(g, \mathbb{S}^{d-1})$ to a representation of $\bar{G}$ on the full Hilbert space.
- Thus any putative noncompact symmetry group must really be a subgroup of a larger compact one!

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The noncompact boson is not finitely generated (and also doesn't have a finite thermal partition function), so it is allowed to have a noncompact symmetry.
Conjecture: CFTs with discrete spectra and a unique stress tensor are always finitely generated.

Conclusion

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- There must be objects that carry all gauge charges
- Global symmetries are probably always compact

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Thanks!