Plan of the talk

1) Motivation.
2) Symmetry reduced model: vacuum quantization.
3) Test quantum fields on quantum geometries.
4) Extension to other 1+1 scenarios.
5) Other approaches.
6) Discussion and outlook.
Motivation

1) Final state of classical collapse
   - Event (dynamical) horizon
   - Curvature blows up at the origin

2) Semiclassical description
   - Hawking radiation & black hole evaporation
   - Information loss paradox!

3) Quantum gravity?
Among the candidates for QG I will focus on loop quantum gravity

- Nonperturbative, background independent & canonical quantization.

- Real Ashtekar-Barbero (canonical) variables: connection $A$ and densitized densitized triad $E$ (electric field) $q \sim E^2$.

- Robust kinematical description: holonomy-flux algebra.

- Gauge transfs. and spatial diffeomorphism well understood (LOST uniqueness theorem, 2005).

- Dynamics (scalar constraint) studied in different scenarios (vacuum, gauge fixing with matter, Master Constraint, Uniform Discretizations, ...) but still needs further analysis.
Symmetry reduced models

The quantization of mini and midisuperspace models adopting LQG quantization techniques has had/has great success

- loop quantum cosmology: singularity replaced by a quantum bounce.

\[ \partial^2_\phi \Psi = -\hat{\Theta} \Psi \]

- Some models admit an exactly solvable quantum dynamics.
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- Anisotropic and even inhomogeneous models are under study.

★ Bianchi cosmologies.
★ Kantowski-Sachs cosmologies.
★ Polarized Gowdy models.
★ Spherically symmetric spacetimes.
★ 1+1 dilatonic black holes.
Spherically symmetric spacetimes in LQG

1) Spherical symmetry (Bengtsson, Thiemann, Bojowald).

2) Reduced connection $A$ and triad $E$.

3) The reduced Hamiltonian is a linear combination of constraints

$$H_T = H(N) + D_x(N^x) + G_x(\lambda^x) + \text{bound. terms}$$  \hspace{1cm} (1)

4) The constraint algebra shows structure functions. After the redefinition $\{H(N), H(\tilde{N})\} = 0$, it becomes a true Lie algebra.

5) The space of solutions is coordinatized by the mass $M$ of the black hole.
1) Kinematical Hilbert space constructed out of spin networks (spherically symmetric 1d graphs). It admits a consistent representation of $[\hat{H}(N), \hat{H}(\tilde{N})] = 0$.

2) The solutions to the constraints can be computed and the physical Hilbert space constructed.

3) Quantum geometries determined by states $|\vec{k}, M\rangle$ (where $k_j$ codifies the areas of symmetry $A_j = \ell^2_{\text{Pl}} k_j$ and $M$ the mass) normalized to

$$\langle \vec{k}, M | \vec{k}', M' \rangle = \delta_{\vec{k}, \vec{k}'} \delta(M - M').$$  \hspace{2cm} (2)

4) Quantum Dirac observables are identified: $\hat{A}$ and $\hat{M}$. The metric components $\hat{g}_{\mu\nu}$ can be defined as relational observables.

5) The singularity is resolved since $k_j \neq 0$ at any vertex.
Semiclassical geometries

1) Requirements for smooth semiclassical geometries
   a) The number of vertices should be very large.
   b) The sequences of \( \{k_j\} \) must be growing (i.e. \( \Delta k_j > 0 \)) with small separations.
   c) The states \( |\Psi_{\text{phys}}\rangle \) peaked around \( M_0 \) and \( \vec{k}_0 \) (superpositions).

2) Consequences: If \( 4\pi r_i^2 = \ell_{\text{Pl}}^2 k_j \), then \( \Delta r_j \geq \frac{\ell_{\text{Pl}}^2}{2r_j} \).

3) Example: for \( |\Psi_{\text{phys}}\rangle \) with \( \vec{k}_0 \) chosen appropriately, it would provide an effective geometry with a uniform spacing \( r_j = (j + j_H)\Delta \).
1) The effective action of fields becomes discretized on these quantum geometries (assuming the fields to have support only on vertices).

2) The equation of the radial modes on a geometry given by the quantum state \( |\Psi_{\text{phys}}\rangle \) in the exterior of the black hole is

\[
\ddot{\phi}_j - \frac{(\phi_{j+1} - \phi_j)}{\Delta_j^2} + \frac{(\phi_j - \phi_{j-1})}{\Delta_j \Delta_{j-1}} + V_\ell(r_j) \phi_j = 0. \tag{3}
\]

3) For instance, if \( \Delta_j \simeq \Delta \) and \( V_\ell(r_j) \simeq 0 \), then

\[
\omega_n^2 = \frac{2 - 2 \cos(k_n \Delta)}{\Delta^2}, \quad k_n = \frac{2\pi n}{\Delta(N - j_H)}. \tag{4}
\]
Test quantum field theories

1) Casimir effect (Gambini, O. & Pullin, 2015)
   - The discrete geometry motivates a cutoff for high angular momentum $\ell \gg \frac{r}{\Delta}$.
   - Finite stress-energy tensor.
   - The Casimir pressure (in the flat approx.) up to small corrections

$$P = -\frac{d}{dL} [E(L_1) + E(L) + E(L_0 - L)] = -\frac{\pi^2}{480L^4} + \mathcal{O}(\Delta^2), \quad \Delta = \alpha \ell_{\text{Pl}} \lesssim \ell_{\text{Pl}} \quad (5)$$
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2) Hawking radiation (Gambini & Pullin, 2014)

- Trans-Planckian modes at the horizon suppressed. Cutoff $(u_2, j - u_1, j) \geq \ell_{Pl}$ in the 2-point funct. (Agullo, Navarro, Olmo & Parker, 2009).
- Standard Hawking radiation is recovered for $\omega \Delta \ll 1$

$$\langle in | \hat{N}_{n_{out}} | in \rangle = \frac{1}{2\pi} \left( \frac{1}{\exp\{2\pi\omega_1/\kappa\} - 1} - \frac{\kappa^2 \ell_{Pl}}{48\pi^4\omega_1} + \mathcal{O}(\ell_{Pl}\kappa) \right), \quad \kappa = \frac{1}{4GM}. \quad (6)$$
Extensions

1) Reissner-Nordström (Gambini, Mato & Pullin, 2015)

- The physical states are now $|\vec{k}, M, Q\rangle$ (with $k_j$ proportional to the spheres of symmetry, $M$ the mass and $Q$ the charge).
- The classical singularity is absent if one restricts the study the spin networks with $k_j > 0$. Instabilities of test fields?

2) Coupling to a thin shell (Campiglia, Gambini, O. & Pullin, 2016)

- A (semi-reduced but) covariant quantization is admissible. Metric components promoted to well defined parametrized observables.
- The quantization admits semiclassical sectors where (time-dependent and non-singularity) geometries reproduce classical GR very well at low curvature regions.
3) CGHS models: (Corichi, O., Rastgoo, 2016)
   - The physical states are $|\vec{k}, M\rangle$ (with $k_j$ the spectrum of the dilatonic field and $M$ the mass).
   - Physical and geometrical metrics are regular.

4) Gowdy cosmologies with local rotational symmetry: (Martín de Blas, O., Pawlowski, 2015)
   - The physical states are $|\vec{k}, h\rangle$ (with $k_j$ the spectrum of the Killing areas and $h$ is a global physical dof (like the mass $M$ in spherical symmetry).
   - We adopt an improved dynamics scheme (Killing areas and volume are the basic operators).
   - The typical cosmological singularity (zero Killing area) is solved.
   - Nontrivial Schrödinger-like evolution with the spatial volume as time parameter.
Other approaches

1) Kantowski-Sachs models: homogeneous loop quantum cosmology  
   (Ashtekar, Bojowlad, Campiglia, Corichi, Gambini, Joe, pullin, Saini, Singh, ...)

2) Consistent algebra: signature change  (Bojowald, Brahma, Reyes, ...)

3) Master constraint and uniform discretizations  (Campiglia, Dittrich, Gambini, Pullin, Thiemann, ...)

4) Quantum gravity with physical time  (Giesel, Husain, Lewandowski, Pawlowski, Thiemann, ...)

5) Radial gauge: spherical symmetry  (Bodendorfer, Duch, Kamiński, Lewandowski, Swie- 
   zewski, Zipfel, ...)

6) Group field theory: quantum gravity condensate  (Gielen, Oriti, Pranzetti, Sindoni, 
   Wilson-Ewing, ...)
Conclusions

1) Spherically symmetric spacetimes in vacuum in loop quantum gravity are regular and smooth (but fundamentally discrete) semiclassical geometries with GR in the low curvature limit.

2) Several phenomena involving test quantum field theories on quantum geometries have been studied.

3) The quantization has been successfully extended to other scenarios.

4) Quantum gravitational collapse of a null dust shell is currently under study (some advances coming soon).
Outlook

1) A deeper understanding of the quantum dynamics.

2) Extension to nonvacuum models: quantum gravitational collapse.

3) Test quantum field theories (perturbations) on quantum geometries and backreaction not fully understood.

4) Black hole evaporation process:
   a) Typical times of evaporation.
   b) The black hole information loss paradox.

5) Comparison with other approaches.

6) Extension to axisymmetric gravity: spinning black holes.