A Gauge Theory Formulation for Continuous Spin Particles

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Irreducible Representations of the Poincaré Group (Wigner, 1939)

- Massive particles with $m^2 > 0$ and integer or half-integer spin
- Particles with $m^2 < 0$: tachyons
- Massless particles with integer or half-integer helicity
- Massless particles with continuous spin: continuous spin particles (CSP): Characterized by a real parameter ρ

Continuous Spin Particles

- Not found in Nature
- No free quantum theory is known
- No interaction with matter is known
- Super-Poincaré has supersymmetric CSP (Brink et al, hep-th/0205145)
- Perturbative string theory has no CSP (Font et al, 1302.4771)
- No spacetime action for a CSP was known until recently (Schuster & Toro, 1404.0675)

Massless Representations

- Casimir invariants: $P^2 = P^\mu P_\mu$ and $W^2 = W^\mu W_\mu$ (W_μ: Pauli-Lubanski vector)
- Massless particles: choose $P^\mu = (p, 0, 0, p) \Rightarrow$ little group E_2 (non-compact!)
- Generators: $W_0 = W_3$, generator of $SO(2)$, and W_1, W_2 generators of $T(2)$

$$\begin{aligned} W^2|\rho,\lambda> &= -(W_1^2 + W_2^2)|\rho,\lambda> = -\rho^2|\rho,\lambda> \\ W_3|\rho,\lambda> &= \lambda|\rho,\lambda> \end{aligned}$$

- For $\rho^2 = 0 \Rightarrow$ usual helicity states $|0,\lambda>, \lambda = 0, \pm 1/2, 1, \ldots$
- For $\rho^2 > 0 \Rightarrow$ CS representation

$$W_\pm|\rho,\lambda> = (W_1 \pm iW_2)|\rho,\lambda> = \pm i\rho|\rho,\lambda \pm 1>$$

so that $|\rho,\lambda>$ with $\lambda = 0, \pm 1/2, \pm 1, \ldots$ is an infinite dimensional representation

- When $\rho \to 0$ the CS representation reduces to a sum of helicity representations for all spin $\to$ HS massless particles

Cotangent Bundle over Minkowski spacetime (Rivelles, 1607.01316)

- Any field $\Psi(\eta, x)$ depends on x^μ and on a covector η_μ
- Similar to phase space but with no symplectic structure
- $\Psi(\eta, x)$ has a formal power series expansion in η_μ

$$\Psi(\eta, x) = \sum_{s=0}^{\infty} \frac{1}{s!} \eta^{\mu_1} \ldots \eta^{\mu_s} \psi_{\mu_1 \cdots \mu_s}(x)$$

  where each $\psi_{\mu_1 \cdots \mu_s}(x)$ is a completely symmetric and unconstrained tensor field in spacetime
- To buid an action we need an integration procedure which allow us to get meaningful actions for the components of $\Psi(\eta, x)$
- Make use of distributions localized on the η-hyperboloid $\eta^2 + 1 = 0$ (Segal, 0103028)

$$\int d^4\eta \; \delta^{(n)}(\eta^2 + 1) \; \phi[\Psi(\eta, x)]$$

  with $\delta^{(n)}(\eta^2 + 1)$ the n-th derivative with respect to the argument
- After Wick rotation the integral will be proportional to a sum of contracted $\Psi(\eta, x)$ components
- This will lead to an unconventional field equation

CSP Action, (Schuster and Toro,1404.0675)

$$S = \frac{1}{2}\int d^4x\, d^4\eta \left[\delta'(\eta^2+1)(\partial_x\Psi(\eta,x))^2 + \frac{1}{2}\delta(\eta^2+1)\left((\partial_\eta\cdot\partial_x+\rho)\Psi(\eta,x)\right)^2\right]$$

- Localized on the η-hyperboloid and its first neighborhood
- Rigid symmetries:
- Invariant under Lorentz transformations
- Invariant under translations in x^μ but NOT under translations in η^μ
- Invariant under a η^μ dependent translation in x^μ: $\delta x^\mu = \omega^{\mu\nu}\eta_\nu$

- Local symmetries:
- Invariant under the two local transformations

$$\delta\Psi(\eta,x) = [\eta\cdot\partial_x - \frac{1}{2}(\eta^2+1)(\partial_\eta\cdot\partial_x+\rho)]\epsilon(\eta,x) + \frac{1}{4}(\eta^2+1)^2\chi(\eta,x)$$

- Local transformations are reducible (Rivelles, 1408.35760)

$$\delta\epsilon = \frac{1}{2}(\eta^2+1)\Lambda(\eta,x),$$
$$\delta\chi = (\partial_\eta\cdot\partial_x+\rho)\Lambda(\eta,x)$$

leaves the previous local transformations invariant.

Field Equation

$$\delta'(\eta^2+1)\left(\Box_x\Psi - \eta\cdot\partial_x(\partial_\eta\cdot\partial_x+\rho)\Psi + \frac{1}{2}(\eta^2+1)(\partial_\eta\cdot\partial_x+\rho)^2\Psi\right) = 0$$

- Unusual structure: localized on the η-hyperboloid and its first neighborhood
- η^μ is a constrained variable: $\eta^2+1=0$
- Hard to work: $\partial\eta^\mu/\partial\eta^\nu$ no longer δ^μ_ν

- First solve for the delta function constraint

$$\delta'(\eta^2 + 1)A(\eta, x) = 0$$

$$A(\eta, x) = \Box_x \Psi - \eta \cdot \partial_x(\partial_\eta \cdot \partial_x + \rho)\Psi + \frac{1}{2}(\eta^2 + 1)(\partial_\eta \cdot \partial_x + \rho)^2 \Psi$$

- Solution: Any $A(\eta, x)$ satisfying $A'(\eta, x) = 0$.
- Simplest solution: $A(\eta, x) = 0$

$$\Box_x \Psi - \eta \cdot \partial_x(\partial_\eta \cdot \partial_x + \rho)\Psi + \frac{1}{2}(\eta^2 + 1)(\partial_\eta \cdot \partial_x + \rho)^2 \Psi = 0$$

- Conventional field equation !
- Now $A(\eta, x) = 0$ everywhere in the cotangent bundle.
- But the dynamics is constrained at the η-hyperboloid
- So at the end we have to return to the η-hyperboloid

Gauge Fixing

- Expand $\Psi(\eta, x)$ around the η-hyperboloid

$$\Psi(\eta, x) = \sum_{n=0}^{\infty} \frac{1}{n!}(\eta^2 + 1)^n \psi_n(\eta, x)$$

and assume that $\psi_n(\eta, x)$ are analytic in η^μ

$$\psi_n(\eta, x) = \sum_{s=0}^{\infty} \frac{1}{s!} \eta^{\mu_1} \dots \eta^{\mu_s} \psi_{n,\mu_1 \cdots \mu_s}(x)$$

- The parameters of the local transformation ϵ, χ and Λ can also be expanded as above
- The local transformations reduce to

$$\delta\psi_n = (1-n)\eta \cdot \partial_x \epsilon_n - \frac{1}{2} n(\partial_\eta \cdot \partial_x + \rho)\epsilon_{n-1} + \frac{1}{4} n(n-1)\chi_{n-2}$$

- Local symmetry parameters transform as

$$\delta\epsilon_n = \frac{1}{2} n \Lambda_{n-1},$$
$$\delta\chi_n = 2\eta \cdot \partial_x \Lambda_{n+1} + (\partial_\eta \cdot \partial_x + \rho)\Lambda_n$$

- Λ symmetry allows to gauge away all ϵ_n except ϵ_0
- χ symmetry allows to gauge away all ψ_n except ψ_0 and ψ_1

$$\delta\psi_0 = \eta \cdot \partial_x \epsilon_0,$$
$$\delta\psi_1 = -\frac{1}{2}(\partial_\eta \cdot \partial_x + \rho)\epsilon_0,$$

- Left with ψ_0, ψ_1 and ϵ_0 and gauge transformation

$$\Psi(\eta, x) = \psi_0(\eta, x) + (\eta^2 + 1)\psi_1(\eta, x)$$
$$\delta\psi_0 = \eta \cdot \partial_x \epsilon_0,$$
$$\delta\psi_1 = -\frac{1}{2}(\partial_\eta \cdot \partial_x + \rho)\epsilon_0,$$

- Harmonic gauge: $(\partial_x \cdot \partial_\eta + \rho)\psi_0 + 2\eta \cdot \partial_x \psi_1 = 0$
- Leads to $\Box_x \psi_0 = \Box_x \psi_1 = \Box_x \epsilon_0 = 0$
- Generalized transverse gauge: $(\partial_x \cdot \partial_\eta + \rho)\psi_0 = 0$
- Leads to $(\partial_x \cdot \partial_\eta + \rho)\epsilon_0 = \psi_1 = 0$
- Momentum space: light-cone components $k_+ \neq 0, k_- = k_i = 0, (i = 1, 2)$
- Write components of $\psi_0(\eta, x)$ as

$$\tilde{\psi}_{\underbrace{+\cdots+}_{p \text{ times}}\underbrace{-\cdots-}_{q \text{ times}} i_1 \dots i_n}(k) = \tilde{\psi}_{(+)^p(-)^q(i)^n}(k)$$

- Solution to generalized transverse gauge:

$$\tilde{\psi}_{0(+)^p(-)^q(i)^n} = \left(-\frac{\rho}{ik_+}\right)^q \tilde{\psi}_{0(+)^p(i)^n}, \qquad p, q, n \geq 0$$

Independent components of ψ_0 are $\tilde{\psi}_{0(+)^p(i)^n}$

- Fixing residual gauge transformation

$$\delta\tilde{\psi}_{0(+)^p(-)^q(i)^n} = pik_+\tilde{\epsilon}_{(+)^{p-1}(-)^q(i)^n}, \qquad p, q, n \geq 0$$

- We find that $\tilde{\psi}_{0(-)^q(i)^n}$, $q, n \geq 0$, and hence $\tilde{\psi}_{0(i)^n}$, are gauge invariant
- Possible to gauge away all $\tilde{\psi}_{0(+)^p(-)^q(i)^n}$, $p \geq 1, q, n \geq 0$
- Summarizing, all $+$ components can be gauged away, all $-$ components can be expressed in terms of the i components and all i components are gauge invariant
- The physical degrees of freedom are $\tilde{\psi}_{0i_1 \dots i_n}(k)$
- Any $\psi_0(\eta, x)$

$$\psi_0(\eta, x) = \sum_{s=0}^{\infty} \frac{1}{s!} \eta^{\mu_1} \dots \eta^{\mu_s} \psi_{0,\mu_1 \cdots \mu_s}(x)$$

  can be rewritten as

$$\psi_0(\eta, x) = \sum_{n=0}^{\infty} \frac{1}{n!} (\eta^2 + 1)^n \phi_n^T(\eta, x)$$

  where the components of $\phi_n^T(\eta, x)$ are traceless
- Going back to the η-hyperboloid only $\phi_0^T(\eta, x)$ survives so that the components of ψ_0 are traceless
- Then ψ_0 describes a tower of helicity states with each helicity appearing once
- Are they describing a CSP or a tower of helicity states?

Quartic Casimir Operator

- On the cotangent bundle P_μ is realized as usual but $J_{\mu\nu} = ix_{[\mu}\partial_{x\nu]} + i\eta_{[\mu}\partial_{\eta\nu]}$

$$W^2\Psi = \Big[\eta\cdot\partial_\eta(1+\eta\cdot\partial_\eta)\Box_x - \eta^2\Box_\eta\Box_x - 2\eta\cdot\partial_\eta\,\eta\cdot\partial_x\,\partial_\eta\cdot\partial_x$$
$$+ (\eta\cdot\partial_x)^2\Box_\eta + \eta^2(\partial_\eta\cdot\partial_x)^2\Big]\,\Psi$$

- Using the gauge fixed solution we find

$$W^2\Psi = -\rho^2\Psi$$

- For $\rho \neq 0$ we have a CSP
- For $\rho = 0$ we have a tower of helicity states for all integer helicities
- For $\rho \neq 0$ it as also possible to show the helicity mixing (Rivelles, 1607.01316)

$$W_\pm|\rho,\lambda >= \pm i\rho|\rho,\lambda\pm 1 >$$

SUMMARY

- We explicitly showed that the Schuster-Toro action describes a CSP
- For $\rho = 0$, after integrating over η^μ, it reduces to a sum of Fronsdal actions for massless fields (Rivelles, 1408.3576)
- It can be extended to higher dimensions
- It can extended to include fermions (Bekaert and Najafizadeh, 1506.00973)

- For $\rho = 0$ it provides an alternative formulation for HS theories
- The massive case can be constructed (to appear)
- Can be extended to (A)dS (in progress)

- Can CSPs interact among themselves and with conventional particles???
- No coupling to a background abelian gauge field (Rivelles, 1408.3576)
- Can the $\rho = 0$ case provide an alternative formulation for HS in flat and AdS spaces???
- Can they be used for AdS_4/CFT_3 duality????