Imprints of Dirac Structure in Emergent Gravity and a geometric T-dual avatar

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Outline

1. Emergent Gravity
   - Review and Two Important Theorems
   - Geometry of Emergent Gravity

2. Geometrical T-duality
   - A plausible T-dual avatar

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- Consider a line bundle L over (M, B) whose connection 1-form is denoted by $A = A_\mu(x)dx^\mu$. The curvature F of a line bundle is a closed two-form, i.e., $dF = 0$ and so locally can be expressed as $F = dA$. The line bundle L over (M, B) admits a local gauge symmetry $\mathfrak{B}_L$ which acts on A as well as on the symplectic structure B on the base manifold M :

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- The local gauge symmetry $\mathfrak{B}_L$ demands the curvature $F = dA$ of L to appear only as the combination $\mathcal{F} \equiv B + F$ since the two-form $\mathcal{F}$ is gauge invariant under the Λ-symmetry. Since $d\mathcal{F} = 0$, the line bundle L over (M, B) leads to a “dynamical” symplectic manifold $(M, \mathcal{F})$ if $\det(1 + F\theta) \neq 0$ where $\theta \equiv B^{-1}$.

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Review continued

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- Let us conceive an analogue in general relativity. In GR, the gravitational force gives rise to the deformation of a given Euclidean space (M, g), which results in a "dynamical" Riemannian manifold $(\mathcal{M}, G)$ where $G = g + h$. The equivalence principle then suggests that the "dynamical" Riemannian manifold $(\mathcal{M}, G)$ can always be trivialized in a locally inertial frame where the metric G goes to the original unperturbed one g.

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- Let us conceive an analogue in general relativity. In GR, the gravitational force gives rise to the deformation of a given Euclidean space (M, g), which results in a "dynamical" Riemannian manifold $(\mathcal{M}, G)$ where $G = g + h$. The equivalence principle then suggests that the "dynamical" Riemannian manifold $(\mathcal{M}, G)$ can always be trivialized in a locally inertial frame where the metric G goes to the original unperturbed one g.
- The electromagnetic force is to the deformation of a symplectic manifold what the gravitational force is to the deformation of a Riemannian manifold. The emergent gravity picture implies that these two deformations are isomorphic to each other.

B field transformation : yet another diffeomorphism!!

Theorem

Since B is a symplectic structure on M, it defines a bundle isomorphism $B : TM \to T^*M$ *by* $X \mapsto \Lambda = -\iota_X B$ *where* $X \in \Gamma(TM)$ *is an arbitrary vector field. As a result, the B-field transformation can be written as*

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Example

$U(1)$ gauge symmetry, $A \mapsto A + d\lambda$, is a particular case of the Λ-symmetry for $\Lambda = d\lambda = -\iota_{X_\lambda} B$. $X_\lambda = -\theta(d\lambda)$ is called a Hamiltonian vector field. Since a vector field is an infinitesimal generator of local coordinate transformations,or, a Lie algebra generator of $\mathrm{Diff}(M)$, the B-field transformation can be identified with a local coordinate transformation generated by the vector field $X \in \Gamma(TM)$. Hence Λ-symmetry can be considered on par with diffeomorphism. In the presence of B-fields, there is an enhancement of the underlying local gauge symmetry.

Darboux Theorem in Emergent Gravity

- The situation is similar to GR where the dynamical symplectic manifold $(M, \mathcal{F})$ can be locally trivialized by a coordinate transformation $\phi \in \mathrm{Diff}(M)$ such that $\phi^*(\mathcal{F}) = \phi^*(B + F) = B$. diffeomorphism, $\phi^* = (1 + \mathcal{L}_X)^{-1} \approx e^{-\mathcal{L}_X}$ for $A = -\Lambda = \iota_X B$.

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- In other words, it is always possible to find a local coordinate transformation eliminating dynamical $U(1)$ gauge fields as far as the spacetime admits a symplectic structure. This statement is known as the Darboux theorem or the Moser lemma in symplectic geometry which is the crux of emergent gravity.

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Theorem

Let (M, ω) be a $2n$- dimensional symplectic manifold. For each point $x \in M$, there is a local chart (U, ϕ) where U is an open neigborhood of x, and $\phi : U \to \mathbb{R}^{2n}$ is a diffeomorphism such that $\phi^*(\sum_{i=1}^n dx_i \wedge dy_i) = \omega_{|U}$

Moser Lemma and an application

Lemma

Let M be a compact manifold with symplectic forms ω_0 and ω_1. Suppose that ω_t, $0 \leq t \leq 1$, is a smooth family of symplectic forms joining ω_0 to ω_1 with cohomology class $[\omega_t]$ independent of t. Then there exists an isotopy $\rho : M \times \mathbb{R} \to M$ such that $\rho_t^*\omega_t = \omega_0$, $0 \leq t \leq 1$.

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Example

There always exists a coordinate transformation ϕ whose pullback maps $\omega' = \omega_1 = \omega + dA$ to ω, i.e., $\phi : y \mapsto x = x(y)$ is such that

$$\frac{\partial x^\alpha}{\partial y^a} \frac{\partial x^\beta}{\partial y^b} \omega'_{\alpha\beta}(x) = \omega_{ab}(y).$$

For a symplectic manifold $(M, \omega_1 = B + F)$ one can always find a local coordinate chart $(U; y^1, \cdots, y^{2n})$ centered at $p \in M$ and valid in the nghbd U s.t. $\omega_0(p) = \frac{1}{2} B_{ab} dy^a \wedge dy^b$, B_{ab} is a const symplectic matrix.

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$$[X+\xi, Y+\eta]_C^{H=dB} = [X,Y] + \mathcal{L}_X\eta - \mathcal{L}_Y\xi - \tfrac{1}{2}d(\iota_X\eta - \iota_Y\xi) - i_X i_Y H,$$

Generalised Geometry in Emergent Gravity

- *Diff* (M) symmetry of $TM \to$ the symmetry of E.
  Let $\phi : M \to M$ be a diffeomorphism of the base manifold M.
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- The symmetries of E, i.e., the endomorphisms of E (the group of orthogonal Courant automorphisms of E), are the composition of a diffeomorphism Φ of $E = TM \oplus T^*M$ and a B-field transformation

$$e^B : E \to E, \qquad e^B(X + \xi) = (1 + B)(X + \xi) = X + \xi + \iota_X B,$$

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- The group of orthogonal Courant automorphisms of $TM \oplus T^*M$ is a semi-direct product of *Diff*(M) and Ω^2_{closed}, i.e. $Diff(M) \ltimes \Omega^2_{closed}$.

Courant automorphism

Diffeomorphism between two different metrics

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- In terms of local coordinates $(g + \kappa B')_{\alpha\beta}(x) = \frac{\partial y^a}{\partial x^\alpha} \Big(h_{ab}(y) + \kappa B_{ab}(y) \Big) \frac{\partial y^b}{\partial x^\beta}$ where $B' = B + \mathcal{L}_X B$ and

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$$h_{ab}(y) = \frac{\partial x^\alpha}{\partial y^a} \frac{\partial x^\beta}{\partial y^b} g_{\alpha\beta}(x)$$
- Local equivalence between symplectic structures $\rightsquigarrow$ diffeomorphic equivalence between two different DBI metrics (Cornalba)

$$\int d^{p+1}x \sqrt{\det\big(g(x) + \kappa(B + F)(x)\big)} = \int d^{p+1}y \sqrt{\det\big(h(y) + \kappa B(y)\big)}$$

Bilinear non-degenerate pairing

Let E and F be linear spaces of dims. m and n respectively, endowed with a bilinear non-degenerate pairing $(,) : E \times F \to \mathbb{R}$, and consider the total space $(F \times E, \langle , \rangle_+)$.
Let E be a linear space (of dim m), $F = E^\star$ and let $(,)$ be the duality pairing of E and $E^\star$. Suppose M is a smooth oriented (compact) m-manifold, $F = \Lambda^k(M)$, i. e. the space of all k-forms, on M and $E = \Lambda^{m-k}(M)$. Now consider the nondegenerate bilinear pairing $(,) : \Lambda^k(M) \times \Lambda^{m-k}(M) \to \mathbb{R}$, $(\alpha, \beta) = \int_M (\beta \wedge \alpha)$ $\forall \alpha, \beta$ s.t. $\alpha \in \Lambda^k(M)$ and $\beta \in \Lambda^{m-k}(M)$. Poincare duality theorem $\Rightarrow$ we effectively identify the dual of F with E. The next step is to associate to $(,)$, the non-degenerate symmetric, bilinear pairing $\langle , \rangle_+$ $\forall \left(f^1, e^1\right), \left(f^2, e^2\right) \in F \times E$.

$$\left\langle \left(f^1, e^1\right), \left(f^2, e^2\right) \right\rangle_+ = \frac{1}{2}\left[\left(f^1, e^2\right) + \left(f^2, e^1\right)\right]$$

Dirac Structure continued.

Theorem

Let F and E be linear spaces, and let $(,) : F \times E \to \mathbb{R}$ be a non-degenerate bilinear pairing and consider a subspace $\mathcal{D} \subset (F \times E, \langle , \rangle_+)$. The orthogonal complement of $\mathcal{D}$, denoted by $\mathcal{D}^\perp$, wrt $\langle , \rangle_+$, is given by:
$\mathcal{D}^\perp = \left\{ \left(\bar{f}, \bar{e} \right) \in F \times E \middle| \left\langle (f, e), \left(\bar{f}, \bar{e} \right) \right\rangle_+ = 0, \forall (s, \alpha) \in \mathcal{D} \right\}.$

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Theorem

Let F and E be linear spaces (of finite dims), endowed with a bilinear nondegenerate pairing (,) and consider the total space $(F \times E, \langle,\rangle_+)$. The linear subspace $\mathcal{D} \subset (F \times E, \langle,\rangle_+)$ is a DS if $\mathcal{D} = \mathcal{D}^\perp$.

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Example

Let E be a linear space of dim m, and let $E^\star$ be the algebraic dual of E and consider the linear maps $A : E \to E^\star$ and $B : E^\star \to E$ respectively. The maps A and B are skew-symmetric maps if and only if their graphs, are DS.

Dirac Manifold

Theorem

Let M be a finite dim. smooth manifold. We define symmetric and skew-symmetric operations on the vector bundle $\mathbb{T}^{big}M := TM \oplus T^*M$ *over M as* $\langle X \oplus \xi,\ Y \oplus \eta \rangle_+ := \frac{1}{2}\{\xi(Y) + \eta(X)\} \in \mathbb{C}^\infty(M)$ *and* $[X \oplus \xi,\ Y \oplus \eta]_C := [X, Y] \oplus (\mathcal{L}_X\eta - \mathrm{i}_Y d\xi) \in \Gamma^\infty(M, \mathbb{T}^{big}M)$ $\forall X \oplus \xi, Y \oplus \eta \in \Gamma^\infty(M, \mathbb{T}^{big}M)$. *A subbundle $\mathcal{D} \subset \mathbb{T}^{big}M$ is called a Dirac structure on M if*

1. $\langle \cdot, \cdot \rangle_+|_{\mathcal{D}} \equiv 0$;
2. *$\mathcal{D}$ has rank equal to dimM;*
3. $[\Gamma^\infty(M, \mathcal{D}), \Gamma^\infty(M, \mathcal{D})]_C \subset \Gamma^\infty(M, \mathcal{D})$.

M together with DS $\mathcal{D} \subset \mathbb{T}^{big}M$ is called a Dirac manifold, denoted by $(M, \mathcal{D})$. In addition to the natural pairing $\langle \cdot, \cdot \rangle_+$, skew-symmetric pairing can be defined. $\langle X \oplus \xi,\ Y \oplus \eta \rangle_- := \frac{1}{2}\{\xi(Y) - \eta(X)\} \in \mathbb{C}^\infty(M)$. *The orthogonal complement of a subbundle* $\mathcal{D} \subset (\mathbb{T}^{big}, \langle, \rangle_+)$, *is* $\mathcal{D}^\perp = (Y, \eta) \in \mathbb{T}^{big}M | \langle (X, \xi), (Y, \eta) \rangle_+ = 0, \forall (X, \xi) \in \mathcal{D}$

Dirac Manifold in Emergent Gravity

Theorem

Let M be a smooth m-manifold and let $\mathbb{T}^{big}M$ be the big tangent bundle of M. The subbundle $\mathcal{D} \subset \left(\mathbb{T}^{big}M, \langle , \rangle_+\right)$ is a Dirac structure if $\mathcal{D} = \mathcal{D}^\perp$

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Example

Suppose that M is a symplectic manifold with a symplectic form ω. that induces the bundle map $\omega^{\flat} : \mathfrak{X}(M) \longrightarrow \Omega^{1}(M), \qquad X \longmapsto \iota_{X}\omega$.
One can obtain the subbundle $\mathrm{graph}\,(\omega^{\flat})$ in $\mathbb{T}^{big}M$ as $\mathrm{graph}\,(\omega^{\flat})_{p} := \{\, X_p \oplus \mathrm{i}_{X_p}\omega_p \in T_pM \oplus T_p^{*}M \,|\, X_p \in T_pM \,\} \quad (p \in M)$ and can verify that the $\mathrm{graph}(\omega^{\flat})$ satisfies all the three conditions for $(M, \mathrm{graph}\,(\omega^{\flat}))$ to be a Dirac manifold. Similarly, any symplectic manifold M defines a Dirac structure on M. In emergent gravity, we have a closed B-field and e^{B} is an automorphism of the Courant bracket iff $dB = 0$, so naturally it induces a bundle isomorphism of this kind and hence (M, B) defines a Dirac manifold.

Outline

Topological Content in T-duality

Bouwknegt, Evslin, Mathai = BEM

- In the low energy limit of type II strings, the bosonic field content are a metric g, closed 3-form H and dilaton φ that satisy modified Einstein equations. Surprsingly these set of eqns. possess a symmetry, namely T-duality that is not found in the ordinary Einstein eqns. and this symmetry relates spaces X, $\hat{X}$ which are torus bundles over a common base space M and can be characterised by an interchange of Chern classes between the torus bundles with topological data associated to the closed 3-form H.

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- T-duality can be seen as an isomorphism between Courant algebroids $E, \hat{E}$ associated to the spaces $X, \hat{X}$. Leaving aside the dilaton φ, the field content (g, H) defines a generalised metric on the Courant algebroid E. Then it is possible to understand the T-duality of the type II string equations as an isomorphism of generalised metrics.

Towards a T-dual Emergent Gravity

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- Let us simplify the discussion by considering T-duality to act in one direction only, *i.e.* T-dualizing on a circle S^1, Consider E to be an oriented S^1-bundle over M characterized by its first Chern class $c_1(E) \in H^2(M, \mathbb{Z})$, in presence of non-trivial H-flux $H \in H^3(E, \mathbb{Z})$.

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- We argue that the T-dual of E is again an oriented S^1-bundle over M, denoted by $\hat{E}$, supporting H-flux $\hat{H} \in H^3(\hat{E}, \mathbb{Z})$, such that $c_1(\hat{E}) = \pi_* H\,, \qquad c_1(E) = \hat{\pi}_* \hat{H}\,$, where $\pi_* : H^k(E, \mathbb{Z}) \to H^{k-1}(M, \mathbb{Z})$ and similarly $\hat{\pi}_*$, are pushforward maps.

T-duality expression and Gysin Sequence

Duality

$$H = H_{(3)} + A \wedge H_{(2)}\,, \qquad \hat{H} = H_{(3)} + \hat{A} \wedge dA$$

with

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The reason for the duality is understood in the following way, for an oriented S^k-bundle E, we've a long exact sequence in cohomology

$$\cdots \to H^k(M,\mathbb{Z}) \xrightarrow{\pi^*} H^k(E,\mathbb{Z}) \xrightarrow{\pi_*} H^{k-1}(M,\mathbb{Z}) \xrightarrow{F\cup} H^{k+1}(M,\mathbb{Z}) \xrightarrow{\pi^*} \cdots\,.$$

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Example

Here is an example that deals with our scenario. Consider the $k = 3$ segment of this sequence.

- For any H-flux $H \in H^3(E, \mathbb{Z})$ we've an associated element $\hat{F} = \pi_* H \in H^2(M, \mathbb{Z})$, and moreover, $F \cup \hat{F} = 0$ in $H^4(M, \mathbb{Z})$. Now, let $\hat{E}$ be the S^1-bundle associated to $\hat{F}$. Reversing the roles of E and $\hat{E}$ in the Gysin sequence, we see that since $F \cup \hat{F} = \hat{F} \cup F = 0$, $\exists$ an $\hat{H} \in H^3(\hat{E}, \mathbb{Z})$ s.t. $\hat{\pi}_* \hat{H} = F$, where $\hat{H}$ is unique up to an element of $\pi^* H^3(M, \mathbb{Z})$. The transformation $(E, H) \to (\hat{E}, \hat{H})$, for a particular choice of $\hat{H}$, is precisely what is T-duality for us.

Proposal to construct $(E, H) \to (\hat{E}, \hat{H})$

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- The ambiguity in $\hat{H}$, upto an element in $\pi^* H^3(M, \mathbb{Z})$, is fixed by requiring that T-duality should act trivially on $\pi^* H^3(M, \mathbb{Z})$, *i.e.* T-duality shouldn't affect H-flux which is completely supported on M.

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- Since H and $\hat{H}$ live on different spaces, in order to compare them we have to pull them back to the correspondence space, which in general is the fibered product $E \times_M \hat{E} = \{(x, \hat{x}) \in E \times \hat{E} \mid \pi(x) = \hat{\pi}(\hat{x})\}$, which is both an $\hat{S}^1$-bundle over E, as well as an S^1-bundle over $\hat{E}$.

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  - Study the interplay between gerbes, topological T-duality and automorphisms of emergent gravity. (work in progress with Hull)

For Further Reading I

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- Dmitriy. M. Belov, Chris M. Hull and Ruben Minasian, *T-duality, Gerbes and Loop Spaces*, arXiv/0710.5151.
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