Effective FRW Radiation Dominated Era

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Introduction

- Quantum Cosmology in Ashtekar variables with EM radiation is explored with the Geometric tools of quantum mechanics
- Newly developed strategy to determine the dynamics of quantum correlations is presented
- We find solutions to the effective equations and find a bounce without loop regularization
- We also find an effective action in covariant form

Theoretical Background

- Geometrical Formulation of QM [Ashtekar, Schilling]
- Time evolution of $F = \left\langle \hat{F} \right\rangle$ is

$$\frac{dF}{dt} = \frac{1}{i\hbar} \left\langle \left[\hat{F}, \hat{H} \right] \right\rangle = \{F, H_Q\}$$

- Quantum Hamiltonian $H_Q = \left\langle \hat{H} \right\rangle$
- Effective Equations of Motion for QM [Bojowald, Skirzewski]
  - Classical variables $x^i = \left\langle \hat{x}^i \right\rangle$ satisfies $\left\{ x^i, x^j \right\} = \epsilon^{ij}$
  - Quantum variables $G^{i_1 \cdots i_n} = \left\langle (\hat{x}^{(i_1} - x^{(i_1}) \dots (\hat{x}^{i_n)} - x^{i_n)}) \right\rangle$

$$\left\{ x^i, G^{j_1 \cdots j_n} \right\} = 0 \quad \left\{ G^{i_1 \cdots i_n}, G^{j_1 \cdots j_m} \right\} \neq 0$$

- In terms of this variables

$$H_Q = \sum_n \frac{1}{n!} H_{,i_1 \dots i_n} \, G^{i_1 \dots i_n}$$

and the equations of motion

$$\dot{x}^i = \left\{ x^i, H_Q \right\} \quad \dot{G}^{i_1 \dots i_n} = \left\{ G^{i_1 \dots i_n}, H_Q \right\}$$

Semiclassical Approximation

- Let $G^{i_1 \dots i_n} \propto \hbar^{n/2}$ [Bojowald, Skirzewski]. At order $\hbar$

$$H_Q = H + \frac{1}{2} H_{,ij}\, G^{ij}$$

- The equations of motion

$$\dot{x}^i = \left\{x^i, H\right\} + \frac{1}{2}\left\{x^i, H_{,jk}\right\} G^{jk}$$

$$\dot{G}^{ij} = \frac{1}{2} H_{,kl} \left\{G^{ij}, G^{kl}\right\} = H_{,kl} \left(\epsilon^{ik} G^{jl} + \epsilon^{jk} G^{il}\right)$$

- Evolution of $G^{ij} \to \mathcal{L}_{X_H} G^{ij} = 0$
- Evaluating the quantum variables on the phase space, *i.e.* $G(x^i)$, allow us to propose solutions

$$G^{ij} = \sum_{A,B} X^i_{F^A} X^j_{F^B} G_{AB}$$

Effective Dynamics

- In the effective sub-space $H_{eff} = H_Q|_{G=G(x^i)}$. Corrected Poisson Bracket [Skirzewski]

$$\left\{x^i, x^j\right\}_{eff} = \epsilon^{ij}(1+\kappa)$$

With $\kappa := \kappa(x^i) \propto \hbar$

- Requiring $\left\{x^i, H_Q\right\} = \left\{x^i, H_{eff}\right\}_{eff}$ we obtain

$$\kappa\epsilon^{ij} H_{,j} + \frac{1}{2} H_{,jk} \left\{x^i, G^{jk}\right\} = 0$$

- Corrected Jacobi identity

$$\{q, \dot{p}\} - \{p, \dot{q}\} + \{H_{eff}, \kappa\} = 0$$

- Uncertainty relations in the effective sub-space

$$G^{qq} G^{pp} - (G^{qp})^2 = \frac{\hbar^2}{4}$$

The Radiation Dominated Universe

- Spatially flat universe with usual cosmological symmetries.
- Radiation pressure is $p = \rho/3$. Energy density $\rho = (C_r)^2 a^{-4}$
- Friedmann equation $a^2 \dot{a}^2 = (C_r)^2$ with solution $a(t) = \sqrt{2C_r t}$
- Ashtekar variables in cosmology [Bojowald]

$$A_i^a = c\delta_i^a \quad \rightarrow \quad c = \gamma \dot{a}$$
$$E_a^i = p\delta_a^i \quad \rightarrow \quad p = a^2$$

- c and p satisfies: $\{c, p\} = 8\pi G\gamma/3$
- The Hamiltonian constraint

$$H = -c^2\sqrt{p} + ((C_r)^2/\sqrt{p}) = 0$$

- Provide equations of motion

$$\dot{p} = \{p, H\} = 2c\sqrt{p}$$

$$\dot{c} = \{c, H\} = -\frac{c^2}{2\sqrt{p}} - \frac{1}{2}\frac{(C_r)^2}{p^{3/2}}$$

with solutions $p = 2C_r t$ and $c = (C_r/2t)^{1/2}$

Quantum Cosmology: The Problem of Time

- Physics doesn't depend on time coordinate [Isham].
- The Hamiltonian constraint [S. Alexander, M. Bojowald, A. Marcianò and D. Simpson] $E = \langle|\vec{E}|\rangle/\sqrt{p}$

$$H = c^2\sqrt{p} - E^2/\sqrt{p} = \frac{1}{\sqrt{p}}(c\sqrt{p} + E)(c\sqrt{p} - E) = 0$$

- We Introduce the field A conjugated to $E \to \{A, E\} = 1$
- In QG $\frac{d}{dt}F(c, p, A, E) = 0$ but using an electric time $t \to A$

$$0 = \left\{F, H_Q\right\} + \frac{\partial F}{\partial t} = \frac{1}{\sqrt{p}}(c\sqrt{p}+E)\left(\left\{F, (c\sqrt{p}-E)\right\} + \frac{\partial F}{\partial A}\right)$$

- where A grows respect $t \to A(t) = (8Et)^{1/2}$
- The new classical Hamiltonian is $H_A = -c\sqrt{p}$. The negative of E generates the equations of motion

$$\frac{dp}{dA} = \sqrt{p}; \quad \frac{dc}{dA} = -\frac{c}{2\sqrt{p}} \longrightarrow p(A) = \frac{A^2}{4}; \quad c(A) = \frac{k}{A}$$

Quantum Cosmology: Quantum Equations

- The quantum Hamiltonian of order $\hbar$

$$E_Q = c\sqrt{p} + \frac{1}{2\sqrt{p}} G^{cp} - \frac{1}{8} \frac{c}{p^{3/2}} G^{pp} + \mathcal{O}(\hbar^2)$$

- The negative of E_Q provide the corrected equations of motion

$$\frac{dp}{dA} = \{p, -E_Q\} = \sqrt{p} - \frac{1}{8p^{3/2}} G^{pp}$$

$$\frac{dc}{dA} = \{c, -E_Q\} = -\frac{c}{2\sqrt{p}} + \frac{1}{4p^{3/2}} G^{cp} - \frac{3}{16} \frac{c}{p^{5/2}} G^{pp}$$

Quantum Cosmology: Effective Equations

- Construction of quantum corrections.
- Conditions on the correction terms (Effective dynamics, Jacobi identity, uncertainty relations).
- We obtain the quantum correction terms

$$G^{cc} = \frac{E^{-2/3}\alpha c^2}{4p} - \beta\frac{c}{p} + \frac{E^{2/3}}{\alpha p}\left(\frac{\hbar^2}{4} + \beta^2\right)$$

$$G^{cp} = -\frac{E^{-2/3}\alpha c}{2} + \beta$$

$$G^{pp} = E^{-2/3}\alpha p$$

  with α and β integration constants of order $\hbar$.
- Finally, the effective equations of motion are

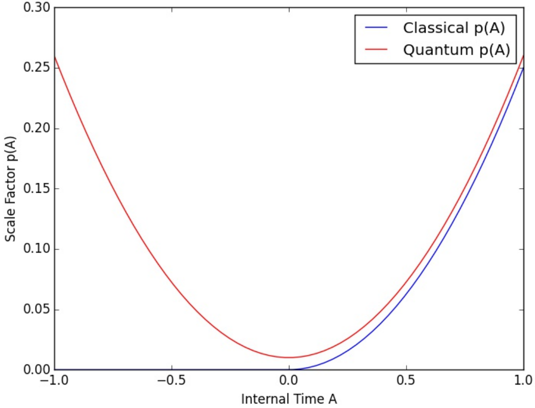
$$\frac{dp}{dA} = \sqrt{p} - \frac{E^{-2/3}}{8\sqrt{p}}\alpha$$

$$\frac{dc}{dA} = -\frac{c}{2\sqrt{p}} - \frac{5cE^{-2/3}}{16p^{3/2}}\alpha + \frac{1}{4p^{3/2}}\beta$$

The Big Bouncing Scenario

- We propose $p(A) = \sum_n p_n(A)\alpha^n$. Employing this in the equation for $p(A)$ we obtain

$$p(A) = \left(\frac{A}{2} + \lambda\alpha\right)^2 + \frac{E^{-2/3}}{4}\alpha$$



Effective Action

- $E_{eff} = c\sqrt{p} - \frac{1}{2\sqrt{p}}\left(\frac{3}{4}c^{1/3}p^{-1/3}\alpha - \beta\right)$
- The corrected Hamiltonian constraint

$$H_{eff} = -c^2\sqrt{p} + \frac{E^2}{\sqrt{p}} - \frac{c}{\sqrt{p}}\left(\frac{3}{4}c^{1/3}p^{-1/3}\alpha - \beta\right)$$

- The corrected $c(t)$ in terms of metric variables

$$c = \dot{a} + \frac{1}{2a^2}\left(\left(\frac{\dot{a}}{a^2}\right)^{1/3}\alpha - \beta\right)$$

- The effective Lagrangian

$$\mathcal{L}_{eff} = -\frac{a^3 R}{6} + \frac{E^2}{a} - \frac{7}{4}a^{-1/3}\left(\frac{\dot{a}}{a}\right)^{4/3}\alpha$$

- Effective Lagrangian for GR

$$\mathcal{L}_{eff} = \frac{\sqrt{g}R}{6} - \frac{\langle\vec{E}^2\rangle}{\sqrt{g}} - \frac{7}{4}\sqrt{g}^{-\frac{1}{9}}\alpha\left(-\frac{R}{12} \pm \frac{1}{12}\sqrt{6R_{\mu\nu}{}^{\lambda\kappa}R_{\lambda\kappa}{}^{\mu\nu} - R^2}\right)^{\frac{2}{3}}$$

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